Want to know $\hat{\theta}$

$\hat{\theta} \overset{iid}{\sim} f(X_1, \dots, X_n)$

say $\theta = \mu$, $\hat{\theta} = \bar{X}$

$E[\hat{\theta}] = E[\bar{X}] = \mu$

Chapter 10

Significance level

P(type I error)
= P(reject H_0 | H_0 true)
= α

Test statistic

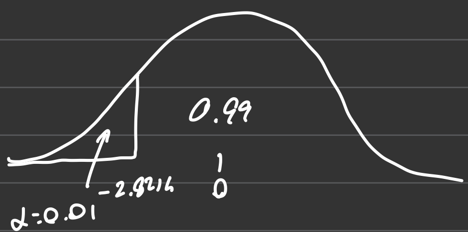
$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$$Z \sim n(0,1)$$

$$RR = \{Z <$$

$H_0: \mu \geq 2$
$H_1: \mu < 2$

$n = 10, \quad \bar{X} = 1.8, \quad s = 0.15$



$$\frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{10}}} \sim t_9$$

$$t_c = \frac{1.8 - 2}{\frac{0.15}{\sqrt{10}}} = -4.22$$

$\therefore$ reject H_0

Finding sample size

type I error

type II

or σ ?

would we know this?

$$n = \left\lceil \frac{(z_\alpha + z_\beta)^2 \times s^2}{(\mu_T - \mu_0)^2} \right\rceil$$

true val

"Most conservative value of x" = value for x with max n

$p = 0.5$ if bi/bernoulli

$X \sim$ an obs. rhino is species A

exhaustive

$\alpha = 5\%$

$H_0: \pi_A = \pi_B = \frac{1}{2}$

$H_1: \pi_A < 0.5$

$n = 11, \quad p = \frac{4}{11}$

$$\bar{X} \sim n\left(0.5 \ , \ \frac{0.5(1-0.5)}{n}\right)$$

$$P(\bar{X} < \phi_{min}) = 0.025$$

$$P\left(\frac{\bar{x} - 0.5}{\sqrt{\frac{0.5(1-0.5)}{n}}} < \underline{\phi_{min} - 0.5}\right) = 0.025$$

ϕ_{mi}

Diff in means

- σ_x, σ_y known
- μ_x, μ_y known

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

- μ_x, μ_y unknown
- $\sigma_x^2 = \sigma_y^2$ but unknown

$$\frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{S_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{(n_x + n_y - 2)}$$

$$S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$$

- μ_x, μ_y unknown
- $\sigma_x \neq \sigma_y$ unknown
- $n_x, n_y < 30$

$$\frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}}} \sim t_v$$

$$v = \left\lfloor \frac{\left(\frac{S_x^2}{n_x} + \frac{S_y^2}{n_y}\right)^2}{\left(\frac{\left(\frac{S_x^2}{n_x}\right)^2}{n_x + 1} + \frac{\left(\frac{S_y^2}{n_y}\right)^2}{n_y + 1}\right)} \right\rfloor$$

$\bar{X} = 9$ $\quad s = 6.424..$ (A)

$P(\bar{X} < \phi_{min}) = 0.05$

$P\left(t_{11} < \dfrac{\phi_{min} - 8.25}{\frac{5.327}{\sqrt{12}}}\right) = 0.05$

$\dfrac{\phi - 8.25}{\frac{5.327}{\sqrt{12}}} = \pm 1.796$

$\phi_{min} = 5.669$ (B)

$\phi_{max} = 12.370$ (C)

9.93 $X \sim n(\mu_1, \sigma^2) \Rightarrow \bar{X} = n(\mu_1, \frac{\sigma^2}{n})$

$Y \sim n(\mu_2, 3\sigma^2) \Rightarrow \bar{Y} = n(\mu_2, \frac{3\sigma^2}{m})$

$\bar{W} = 2\bar{X} + \bar{Y}$

$E[W] = 2\mu_1 + \mu_2$

$W \sim n\left(2\mu_1 + \mu_2, \frac{4 \times \sigma^2}{n} + \frac{3 \times \sigma^2}{m}\right)$

$P(W < \phi_{min}) = 0.05$

$\dfrac{\phi_{min} - (2\mu_1 + \mu_2)}{}$ Help Why do they do Θ in memo

8.94.

$$\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$S_p = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$$

$$P\left(t_{n_1+n_2-2} > \frac{(\bar{X}-\bar{Y})-\phi_{max}}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\right) = 1-\alpha$$

$$\frac{(\bar{X}-\bar{Y})-\phi_{max}}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = t_{n_1+n_2-2,\alpha}$$

$$\phi_{max} = -\left(t_{n_1+n_2-2,\alpha U} \times S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} - (\bar{x}-\bar{y})\right)$$

8.95 $\quad \bar{X} = 85.73\dot{3}$ (A) $\quad s = 0.708\ldots$ (B) $\quad n=6$

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$P\left(\chi^2_{0.05L} < \frac{5s^2}{\sigma^2} < \chi^2_{0.05U}\right) = 0.1$$

Why on outside

$$\sigma^2\chi^2_{0.05L} < 5s^2 < \chi^2_{0.05U}\,\sigma^2$$

Variance pooling

$H_0: \frac{\sigma_x^2}{\sigma_y^2} = 1$

$H_1: \frac{\sigma_x^2}{\sigma_y^2} \neq 1$

$F_{stat} = \frac{s_x^2}{s_y^2}$

$F_{crit} = F_{(n_x, n_y), \frac{\alpha}{2}}$

$F_{crit_1} < F_{stat} < F_{crit_2}$

$\therefore$ Fail to reject H_0

Can pool variances

$F_{stat} < F_{crit_1}$ or $F_{stat} > F_{crit_2}$

$\therefore$ reject H_0

Can't pool variance